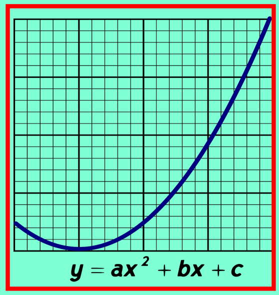
Math 125

Spring 2021

Lecture 16



Class QZ 11

Solve by matrix Method:

$$\begin{cases} x + 2y = 5 \\ 2x - y = 0 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 2 & -1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & -5 & -10 \end{array}\right]$$

$(-2)R1 + R2 \rightarrow R2$ $\qquad$ $R2 \div (-5) \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right] \begin{array}{l} \Rightarrow x = 1 \\ \Rightarrow y = 2 \end{array} \Rightarrow \boxed{(1,2)}$$

$(-2)R2 + R1 \rightarrow R1$

Solve by Subs.

$$\begin{cases} x - y = -4 \\ y = 4x^2 + 1 \end{cases}$$

Non-linear eqn.

$x - (4x^2 + 1) = -4$

$x - 4x^2 - 1 = -4$

$-4x^2 + x - 1 + 4 = 0$

$-4x^2 + x + 3 = 0$

multiply by -1

$4x^2 - x - 3 = 0$

Factor

$(4x + 3)(x - 1) = 0$

$4x + 3 = 0 \qquad x - 1 = 0$

$\boxed{x = \frac{-3}{4}} \qquad \boxed{x = 1}$

$x = 1$

$y = 4x^2 + 1 = 4(1)^2 + 1 = 5$

$\Rightarrow \boxed{(1, 5)}$

$x = \frac{-3}{4}$

$y = 4x^2 + 1 = 4\left(\frac{-3}{4}\right)^2 + 1 = \cancel{4} \cdot \frac{9}{\cancel{16}} + 1$

$= \frac{9}{4} + \frac{4}{4}$

$= \boxed{\frac{13}{4}}$

$\Rightarrow \left(\frac{-3}{4}, \frac{13}{4}\right)$

$\left\{(1,5), \left(\frac{-3}{4}, \frac{13}{4}\right)\right\}$

Solve

$$\begin{cases} x - y = 3 \\ x^2 - xy + y^2 = 13 \end{cases}$$

Isolate x then use Subs. method

$x = 3 + y$

$(3 + y)^2 - (3 + y)y + y^2 = 13$

$(3 + y)(3 + y) - y(3 + y) + y^2 = 13$

$9 + 3y + \cancel{3y} + y^2 - \cancel{3y} - \cancel{y^2} + \cancel{y^2} - 13 = 0$

$y^2 + 3y - 4 = 0$

Factor & Solve each factor

$(y + 4)(y - 1) = 0$

$y + 4 = 0 \qquad y - 1 = 0$

$y = -4 \qquad y = 1$

$y = 1$

$x = y + 3 = 1 + 3 = 4$

$\boxed{(4, 1)}$

$y = -4$

$x = -4 + 3 = -1$

$\boxed{(-1, -4)}$

Final Ans

$\{(4, 1), (-1, -4)\}$

Solve by elimination

$$\begin{cases} 2x^2 + 3y^2 = 35 \\ -2 \quad x^2 - 2y^2 = -14 \end{cases} \Rightarrow \begin{cases} 2x^2 + 3y^2 = 35 \\ -2x^2 + 4y^2 = 28 \end{cases}$$

$$7y^2 = 63$$

$$y^2 = 9 \Rightarrow \boxed{y = \pm 3}$$

$$x^2 - 2(9) = -14$$

$$x^2 - 18 = -14$$

$$x^2 = -14 + 18$$

$$x^2 = 4$$

$$\boxed{x = \pm 2}$$

$(2,3)$, $(-2,3)$

$(2,-3)$ $(-2,-3)$

Variations:

1) Direct

y varies directly as x^2.

$$y = k \cdot x^2$$

2) Inverse

y varies inversely as $\sqrt{x}$

$$y = \frac{k}{\sqrt{x}}$$

K is Constant of Variations

y varies directly as x^2. $\Rightarrow y = K \cdot x^2$

y is 50 when x is 5. $\Rightarrow 50 = K \cdot 5^2$

$50 = K \cdot 25$

Find y when x is 10.

$\boxed{K = 2}$

$y = 2x^2$

$y = 2(10)^2$

$\boxed{y = 200}$

y varies directly as $\boxed{\text{square root of } x}$.

y is 100 when x is 25.

$y = K\sqrt{x}$

$100 = K\sqrt{25}$

Find y when x is 16.

$100 = K \cdot 5$

$\boxed{K = 20}$

$y = 20\sqrt{x}$

$= 20\sqrt{16}$

$= 20 \cdot 4$

$\boxed{y = 80}$

y varies inversely as x

y is 2.5 when x is 4.

Find y when x is 2.

$y = \frac{k}{x}$

$2.5 = \frac{k}{4}$

$k = 4(2.5)$

$\boxed{k = 10}$

$y = \frac{10}{x}$

$y = \frac{10}{2}$ $\boxed{y = 5}$

y varies inversely as cube of x.

y is 10 when x is 4.

Find y when x is 2.

$y = \frac{k}{x^3}$

$10 = \frac{k}{4^3}$

$10 = \frac{k}{64}$

$k = 10(64)$

$\boxed{k = 640}$

$y = \frac{640}{x^3}$

$y = \frac{640}{2^3}$

$= \frac{640}{8}$

$\boxed{y = 80}$

Variations:

1) Direct

$\underbrace{y \text{ varies directly as } x^2}$.

$y = k \cdot x^2$

2) Inverse

$\underbrace{y \text{ varies inversely as } \sqrt{x}}$

$y = \frac{k}{\sqrt{x}}$

k is Constant of Variations

3) Joint

z varies directly as x and inversely as y^2.

$z = \frac{k\,x}{y^2}$

z varies directly as x^2 and inversely as $\sqrt{y}$.

z = 1 when x is 2 and y is 16.

Find z when x is 6 and y is 4.

$z = \frac{k\,x^2}{\sqrt{y}}$ $\quad 1 = \frac{k \cdot 2^2}{\sqrt{16}}$ $\quad 1 = \frac{k \cdot 4}{4}$ $\quad \boxed{k=1}$

$z = \frac{x^2}{\sqrt{y}}$

$z = \frac{6^2}{\sqrt{4}} = \frac{36}{2} = \boxed{18}$

Z varies directly as square root of the sum of x^2 and y^2. $Z = k\sqrt{x^2+y^2}$

Z is 20 when x is 3 and y is 4.

Find Z when $x=6$ and $y=8$.

$20 = k\sqrt{3^2+4^2}$

$20 = k\sqrt{25}$

$20 = k \cdot 5$ $\boxed{k=4}$

$Z = 4\sqrt{x^2+y^2}$

$= 4\sqrt{6^2+8^2} = 4\sqrt{100}$ $\boxed{Z=40}$

Z varies inversely as square root of the difference of x^2 and y^2. $Z = \dfrac{k}{\sqrt{x^2-y^2}}$

Z is 10 when $x=10$ and $y=6$

Find Z when $x=5$ and $y=3$.

$10 = \dfrac{k}{\sqrt{10^2-6^2}}$

$10 = \dfrac{k}{\sqrt{64}}$

$10 = \dfrac{k}{8}$

$\boxed{k=80}$

$Z = \dfrac{80}{\sqrt{x^2-y^2}}$

$Z = \dfrac{80}{\sqrt{5^2-3^2}}$

$Z = \dfrac{80}{\sqrt{16}} = \dfrac{80}{4}$ $\boxed{Z=20}$

Class QZ 12

Hint: Use subs. method

Solve

$$\begin{cases} x - 2y = 0 \\ x^2 + y^2 = 5 \end{cases}$$

$x = 2y$

$x = 2y$

$(2y)^2 + y^2 = 5$

$4y^2 + y^2 = 5$

$5y^2 = 5$

$y^2 = 1$

$\boxed{y = \pm 1}$

$y = 1 \rightarrow x = 2(1) = 2 \Rightarrow (2, 1)$

$y = -1 \rightarrow x = 2(-1) = -2 \Rightarrow (-2, -1)$

$\boxed{\{(2,1), (-2,-1)\}}$

Rational exponents & Radicals

Fraction

Index

Radical

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Radicand

$x^{\frac{2}{3}} = \sqrt[3]{x^2}$

Index = 3

Radicand = x^2

$x^{\frac{3}{4}} = \sqrt[4]{x^3}$

Index = 4

Radicand = x^3

$\sqrt[5]{(x-2)^2}$

$= (x-2)^{\frac{2}{5}}$

index = 5

Radicand = $(x-2)^2$

write using rational exponent

when index is even $\Rightarrow$

Radicand ≥ 0

when no index is given $\Rightarrow$

index $= 2$

$\Rightarrow$ square root

Find domain

$f(x) = \sqrt{x-3}$

No index $\Rightarrow$ index $= 2$ $\Rightarrow$ even index

even root

Radicand ≥ 0 $\quad x-3 \geq 0$ $\quad x \geq 3$

$\boxed{\text{Domain: } [3, \infty)}$

Find the domain

$f(x) = \sqrt[4]{-2x-5}$ "4th root of" $-2x-5$

index $= 4$ $\Rightarrow$ even index $\Rightarrow$ even root

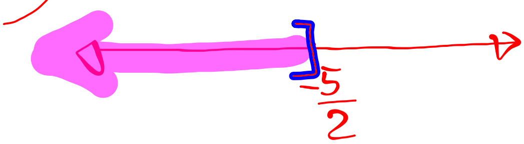
$\Rightarrow$ Radicand ≥ 0

$-2x - 5 \geq 0$

$-2x \geq 5$

$\frac{-2}{-2}x \leq \frac{5}{-2}$

$x \leq -\frac{5}{2}$



Domain $\Rightarrow$ $\left(-\infty, -\frac{5}{2}\right]$

Simplify $\sqrt[3]{x} \cdot \sqrt[4]{x}$

Hint:

$x^m \cdot x^n = x^{m+n}$

$x = x^1$

$\sqrt[3]{x^1} = x^{\frac{1}{3}}$

$\sqrt[4]{x^1} = x^{\frac{1}{4}}$

$= x^{\frac{1}{3}} \cdot x^{\frac{1}{4}}$

$= x^{\frac{1}{3}+\frac{1}{4}}$

$= x^{\frac{7}{12}}$

$= \sqrt[12]{x^7}$

Index = 12

Radicand = x^7

Simplify

$\sqrt[4]{x^3} \cdot \sqrt[8]{x} = x^{\frac{3}{4}} \cdot x^{\frac{1}{8}}$

$\frac{6}{8} + \frac{1}{8} = \frac{7}{8}$

$= x^{\frac{6}{8}} \cdot x^{\frac{1}{8}}$

$= x^{\frac{7}{8}}$

$= \sqrt[8]{x^7}$

index = 8

Radicand = x^7

Simplify

$$\frac{\sqrt{x}}{\sqrt[3]{x}} = \frac{x^{\frac{1}{2}}}{x^{1/3}}$$

$$= x^{\frac{1}{2}-\frac{1}{3}} = x^{\frac{1}{6}}$$

$$= \sqrt[6]{x^1} = \boxed{\sqrt[6]{x}}$$

Index $=6$, Radicand $=x$

Hint

$$\frac{x^m}{x^n} = x^{m-n}$$

$$x^1 = x$$

Some rules:

$$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$$

$$\sqrt{50y} = \sqrt{25 \cdot 2y}$$

$$= \sqrt{25}\,\sqrt{2y}$$

$$= \boxed{5\sqrt{2y}}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

$$\sqrt[3]{\frac{x}{125}} = \frac{\sqrt[3]{x}}{\sqrt[3]{125}}$$

$$= \boxed{\frac{\sqrt[3]{x}}{5}}$$

Work on SG 11

Will upload more SG Soon

Exam 2 is next week

All exams are cumulative.

Class QZ 13

Evaluate

$$\begin{vmatrix} 2 & -5 & 1 \\ 1 & 3 & 0 \\ 3 & -2 & 1 \end{vmatrix}$$